**NTU SSS Economics HE2001**  
**Tutorial 10 (Social Choice)**

1) One procedure for aggregating preferences is to use the **plurality rule** is as follows.

Suppose that there are alternatives.

Given the alternatives, pick the alternative that is the top choice of the largest number of individuals and place it at the top of the social preference ordering. Ties are broken randomly. Then, remove that alternative from every individual’s preference ordering, keeping the same order for the remaining alternatives.[[1]](#footnote-1)

Given the alternatives remaining, pick the alternative that is the top choice of the largest number of individuals and place it at the next highest position of the social preference ordering. Ties are broken randomly. Then, remove that alternative from every individual’s preference ordering, keeping the same order for the remaining alternatives.

Repeat the process until there are no remaining alternatives.

(a) Will the plurality rule social decision mechanism result in a complete and transitive social preference ordering? Explain.

Yes. Notice that the procedure will never stop halfway through. By following this procedure, for any set of individual preferences, we will always arrive at a social preference ordering of all alternatives: . With this social preference ordering, we will be able to state the (social) preference over each pair of alternatives. Furthermore, it is also transitive by the fact that it is already in a rank ordered form.

(b) Does the plurality rule satisfy the pareto condition? Explain.

Yes, by following this procedure, for any set of individual preferences, if they all prefer an option to option , Y can never be above X in the social preference ordering according to the plurality rule. I.e. is socially preferred to .

This is because if is above in the social preference ordering, then it implies that when both were still available, was the top choice by the largest number of individuals. But this contradicts that is preferred to by everyone.

(c) Consider the case of 3 alternatives and a society with 9 individuals with the following preference list:

4 individuals have

3 individuals have

2 individuals have

What is the social ranking according to the plurality rule?

*A is top rank. After removing A, B is top rank.*

*By the plurality rule, the social ranking is .*

(d) Now, suppose we swap the middle 3 individuals’ preferences over B and C such that the preference list becomes:

4 individuals have

3 individuals have

2 individuals have

What is the social ranking according to the plurality rule?

*B is top rank. After removing B, A is top rank.*

*By the plurality rule, the social ranking is .*

(e) Does the plurality rule social decision mechanism satisfy the requirement of independence of irrelevant alternatives? Hint: use parts c) and d).

*Intuitively, independence of irrelevant alternatives means that as long as individual preferences over and remain the same, switches in preferences concerning other (non-relevant) options should not affect the social preferences over and y.*

*Note that no individuals have changed their individual rankings between A and B but the social ranking between A and B has flipped. Hence the plurality rule does not satisfy IIA.*

2) In the lecture, we discussed one possible method of aggregating individual preferences: the Borda count, also known as rank-order voting.

Suppose that there are a finite number of alternatives to choose from and that every individual has complete, reflexive, and transitive preferences. For the time being, let us also suppose that individuals are never indifferent between any two different alternatives but always prefer one to the other.

1. Will the Borda Count social decision mechanism always result in a complete and transitive social preference ordering? Explain.

*Yes, it is complete. By following this procedure, we will always be able to arrive at a set of (weak) social preference relations. This is because since the individual preferences are complete and transitive, we will always be able to compare the sum of the number of points of different alternatives and hence it is complete.*

*It is also transitive. For any set of individual preferences, If and , the number of points of must be less or equal to and the number of points of must be less or equal to . Hence, the number of points of must be less or equal to . Hence, we also have .*

1. Does the Borda Count social decision mechanism defined in this way satisfy the pareto condition? Explain.

*Yes. If everybody ranks x ahead of y, then for each individual, x has less points than y. Then the sum of the points of x must hence be less than the sum of the points of y. Thus, x will be socially preferred to y. This implies that the borda count satisfies the pareto condition.*

(c) Suppose that there are two voters and three candidates, , , and . Suppose that Voter 1 ranks the candidates: first, second, and third. Suppose that Voter 2 ranks the candidates: first, second, and third. What is the Borda count for , and ?

, .

(d) Now suppose that it is discovered that candidate once lifted a beagle by the ears. Voter 1, who has rather large ears himself, is appalled and changes his ranking to first, second, third. Voter 2, who picks up his own children by the ears, is favorably impressed and changes his ranking to first, second, third. What is the Borda count for and ?

, .

(e) Does the Borda Count social decision mechanism satisfy independence of irrelevant alternatives? Hint: use parts d) and e).

*No. In the above example, the ranking of z changed, but nobody changed his mind about whether x was better than y or vice versa. Before the change, x was socially preferred to y. After the change, y was socially preferred to x. This violates IIA as defined (see 1c).*

(f) Do you think that with Borda Voting, voters always have an incentive to truthfully report their full preference rankings? (Hint: provide an example where a voter changes his reported ranking and benefits from it)

*Consider the following example:*

|  |  |  |  |
| --- | --- | --- | --- |
| ***Rank*** | ***Bill*** | ***Bertha*** | ***Bob*** |
| *1* | *X* | *Y* | *Z* |
| *2* | *Y* | *Z* | *X* |
| *3* | *Z* | *X* | *Y* |
| *4* | *W* | *W* | *W* |

*If preferences are reported truthfully, then and Z each get 6 points while gets 12 points. Assume that in this case, and Z are assigned with equal chance.*

*If Bertha reports a different preference as follows:*

|  |  |  |  |
| --- | --- | --- | --- |
| ***Rank*** | ***Bill*** | ***Bertha*** | ***Bob*** |
| *1* | *X* | *Y* | *Z* |
| *2* | *Y* | *W* | *X* |
| *3* | *Z* | *Z* | *Y* |
| *4* | *W* | *X* | *W* |

*Then, and the social choice will be Y according to the Borda vote, which Bertha prefers to say getting X,Y,Z with equal chance. Hence, Bertha will have an incentive to lie about her preference ranking even if everyone else is telling the truth.*

3) Norton and Ralph have a utility possibility frontier that is given by the following equation, (where R and N signify Ralph and Norton respectively).

(a) If we set Norton’s utility to zero, what is the highest possible utility Ralph can achieve? If we set Ralph’s utility to zero, what is the best Norton can do?

The utility possibility frontier gives the set of efficient utilities that can be allocated to each individual under the constraints of the economy. By substituting one’s utility to be 0 in the UPF, we are hence able to get the highest utility the other can obtain.

(b) Plot the utility possibility frontier on a graph.

*This is given by the graph*

Chart, line chart

Description automatically generated

(c) Derive an equation for the slope of the above utility possibility curve.

(d) Suppose that a social planner has a utilitarian social welfare function. What is the utility of Ralph and Norton in his ideal allocation?

*The social planner maximises (utilitarian) such that .*

*The ideal allocation is where the slope of the UPC = the slope of the iso-welfare function.*

*i.e. It satisfies . Hence and .*

(e) Both Ralph and Norton believe that the ideal allocation is given by maximizing an appropriate social welfare function. Ralph thinks that is the best distribution of welfare, and presents the maximization solution to a weighted-sum-of-the-utilities social welfare function that confirms this observation. What was Ralph’s social welfare function?   
(*Hint: What is the slope of Ralph’s social welfare function?*)

*Each of them maximises (weighted sum of utilities) for some unknown and .*

*At the optimal point, the iso-welfare line should be tangent to the utility possibility curve.*

*At , . The slope of the “indifference curve” for a social welfare function is . Since the slope of the “indifference curve” needs to be the same as the slope at the tangency point, we must have Any values of and which satisfy this are a possible social welfare function.*

*One example is .*

(f) Norton, on the other hand, believes that is the best distribution. What is the social welfare function Norton presents?

U*sing the same method, . One example is .*

4)  Suppose the utility possibility frontier for two individuals is given by .

1. Plot the utility possibility frontier.

形状

中度可信度描述已自动生成

1. In order to maximize a “Nietzschean social welfare function,” , what values of would one set? Draw a few isowelfare lines and the social optimal point.

形状, 多边形

描述已自动生成

1. If instead we use a Rawlsian criterion, , what values of would maximize the social welfare function? Draw a few isowelfare lines and the social optimal point.

图表, 折线图

描述已自动生成

1. Suppose that social welfare is given by , what values of would maximize the social welfare function? Draw a few isowelfare lines and the social optimal point.

Slope of utility possibility frontier = slope of iso welfare curve

Which gives . Substituting this into utility possibility frontier, .

The diagram will look similar to the one in the lecture because here social welfare is of the “cobb douglas” form.

**Sample questions**

1) In order to combat environmental change, the government of Fruitland decides to implement a new environmental policy. There are three choices of policies: and , and consumers have to vote on which policy to implement. Amongst the population of 1000, there are three *types* of preference rankings: “Young”, “Middle Aged” and “Old” as illustrated below.

|  |  |  |  |
| --- | --- | --- | --- |
| ***Rank*** | ***Young*** | ***Middle Aged*** | ***Old*** |
| *1* | *Z* | *X* | *Y* |
| *2* | *Y* | *Z* | *X* |
| *3* | *X* | *Y* | *Z* |

There are 300 young consumers, 300 middle aged consumers and 400 old consumers.

Recall Borda voting as discussed in the lecture let us call this *standard* Borda voting.   
In contrast, consider *modified* Borda voting where all rules remain the same, except that the number of points of middle-aged consumers is multiplied by 1.5, while the number of points of young consumers is multiplied by 2.

1. In general, how do you think the modified Borda voting rule will influence the social preference ranking, compared to standard Borda voting? Explain. **(5 marks)**
2. Does the *modified* Borda voting social decision mechanism satisfy the Pareto condition? Explain. **(6 marks)**
3. What will be the policy choice if the government uses i) a standard Borda voting rule, ii) a modified Borda voting rule. **(6 marks)**
4. Discuss why the modified Borda voting rule as above might be more “reasonable” than the standard Borda voting rule in the above scenario. **(6 marks)**

2) While there are many different pareto efficient allocations, a social planner believes that the ideal allocation for Ivy and Joseph is given by maximizing an appropriate social welfare function.

Suppose that the utility possibility frontier of the exchange economy can be expressed as .

1. Explain what the utility possibility frontier is and illustrate it in a graph. **(4 marks)**
2. Suppose that the social planner is *utilitarian*, how much utility will Ivy and Joseph get in the social planner’s ideal allocation? Illustrate this on your graph.  **(6 marks)**
3. Suppose that instead, the social planner has a *weighted-utilitarian* social welfare function. He thinks that the ideal allocation gives . What is the social planner’s social welfare function? Illustrate this on your graph. **(6 marks)**
4. Will every point on the utility possibility frontier be achievable as a competitive equilibrium? Discuss. **(6 marks)**

1. For example, if , and we are removing alternative , then the new preference ordering is . [↑](#footnote-ref-1)